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High-gain FEL Theory

- High-gain FEL: Bonifacio, Pellegrini & Narducci, *Opt. Comm.*, 1984



- SASE 1D: starting from noise
 - 1. Kim, *NIMA*, 1986, coasting beam with energy spread
 - 2. Wang & Yu, *NIMA*, 1986, bunched, monochromatic beam
 - 3. Krinsky, *PRE*, 1999, extend 2, coherent vs. incoherent SASE
- 3D effects
 - 1. Moore, *NIMA*, 1986, gain guiding
 - 2. Kim, *PRL*, 1986, general dispersion relation
 - 3. Yu, Krinsky & Gluckstern, *PRL*, 1990, universal scaling
 - 4. Xie, *PAC95*, 1995, handy fitting formula; 1999, eigenmodes

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New Effects and Developments

- IR SASE demonstration experiments employ short electron bunches bunch density gradient effects: CSE, gain, temporal structure extend 1D theory and improve nonlinear simulation [1]
- Transverse coherence of high-gain FEL ==> mode profile, mode discrimination computation of the fundamental and higher-order modes [3]
- Harmonic generation to reach shorter and multiple coherent wavelengths ==> nonlinear harmonic interaction (simulations indicate large power) thorough 3D analysis, numerical examples [2, 3]

[1,2] Z. Huang, K.-J. Kim, to be published in FEL99 conference proceedings.
[3] Z. Huang, K.-J. Kim, in preparation.

Effects of Bunch Density Gradient

- Better understanding of IR SASE experiments
- Bunch length $>>$ radiation wavelength, $>$ slippage length, but bunch density gradient is important because of the slippage effect
- The role of coherent spontaneous emission (CSE)

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1D Linear Theory

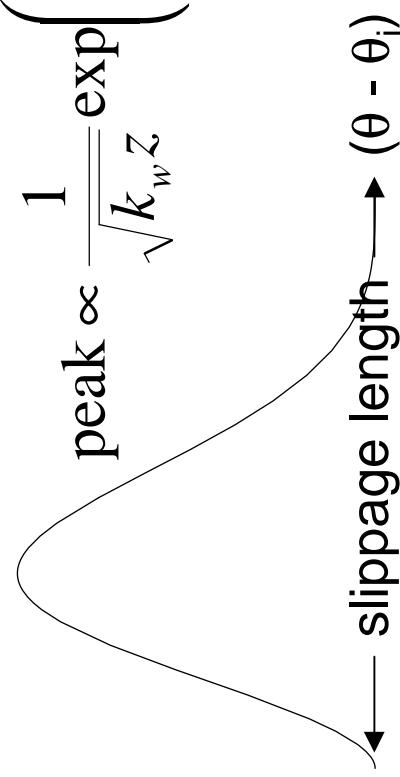
- For an arbitrary phase-space distribution
1D Maxwell-Vlasov equation
slow varying amplitude approximation
==> **field amplitude**

$$E \propto \sum_j e^{-i\theta_j} G(\theta, \theta_j, z)$$

- θ_j is the electron's longitudinal position
- **G is the Green's function for FEL amplification**
vanishing energy spread, ==> Krinsky (1999)
flat-top bunch profile, G looks like

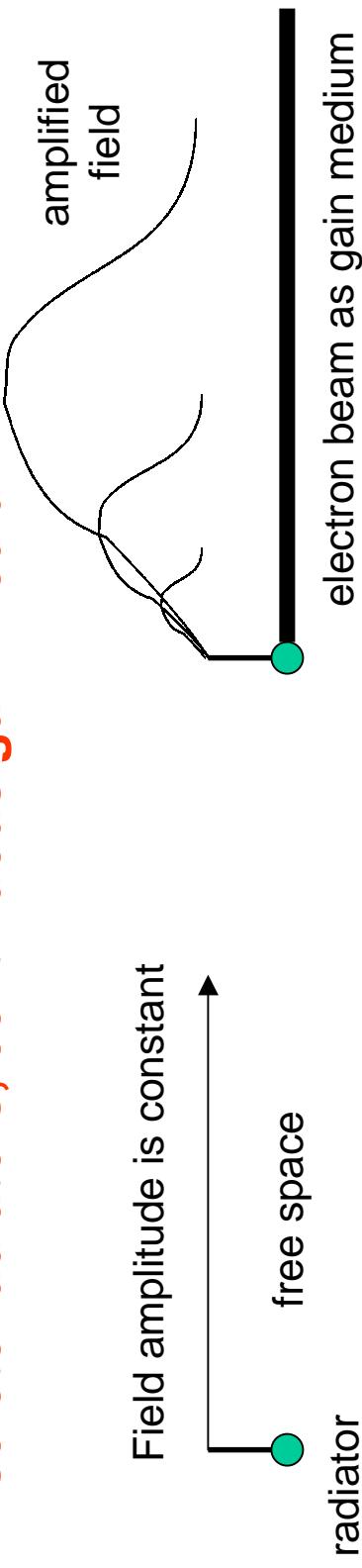
$$\text{peak} \propto \frac{1}{\sqrt{k_w z}} \exp\left(\frac{z}{2L_g}\right)$$

at time z



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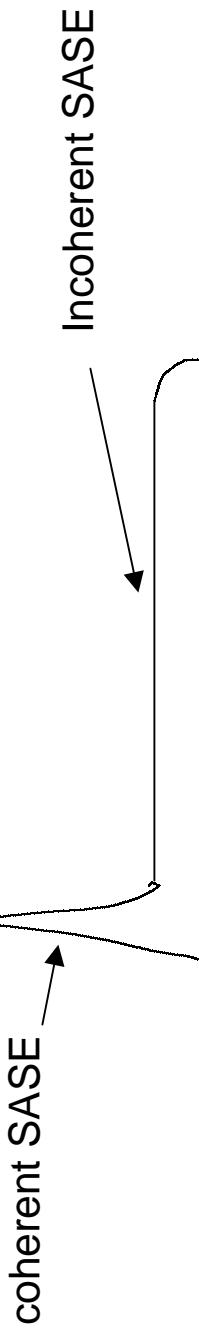
- **Discrete radiators, continuous gain medium**



- SASE intensity for a bunch profile $\chi(\theta) \leq 1$ with maximum line density

$$n_0 \quad I \propto \langle EE^* \rangle \propto n_0 \int d\theta_j \chi(\theta_j) |G|^2 + \frac{n_0^2}{k_r} \left| \int d\theta_j e^{i\theta_j} \frac{d(\chi G)}{d\theta_j} \right|^2$$

- Bunch density gradient => Coherent SASE
- CSE from sharp gradient acts as a macro-charge => spikes



Dispersion Relation

- Complex growth rate λ and radiation detuning ν are related through

$$\lambda - \frac{\nu}{2} + \rho^3 w(\theta, \theta_j) \int d\eta \frac{dV / d\eta}{\lambda - \eta} = 0$$

ρ is the Pierce parameter,
 $V(\eta)$ is the energy spread,

$$w(\theta, \theta_j) = \frac{1}{(\theta - \theta_j)} \int_{\theta_j}^{\theta} \chi(\theta') d\theta' \leq 1 \text{ (gain reduction)}$$

- For a coasting beam, $w(\theta, \theta') = 1 \implies \text{Kim (1986)}$
- For a bunched monochromatic beam $\implies \text{Kaminsky (1999)}$
- **If the bunch shape does not change much over the slippage length**
 $\implies w(\theta, \theta') \approx \chi(\theta)$
 $\implies \text{gain} \propto (\text{local bunch density gradient})^{1/3}$

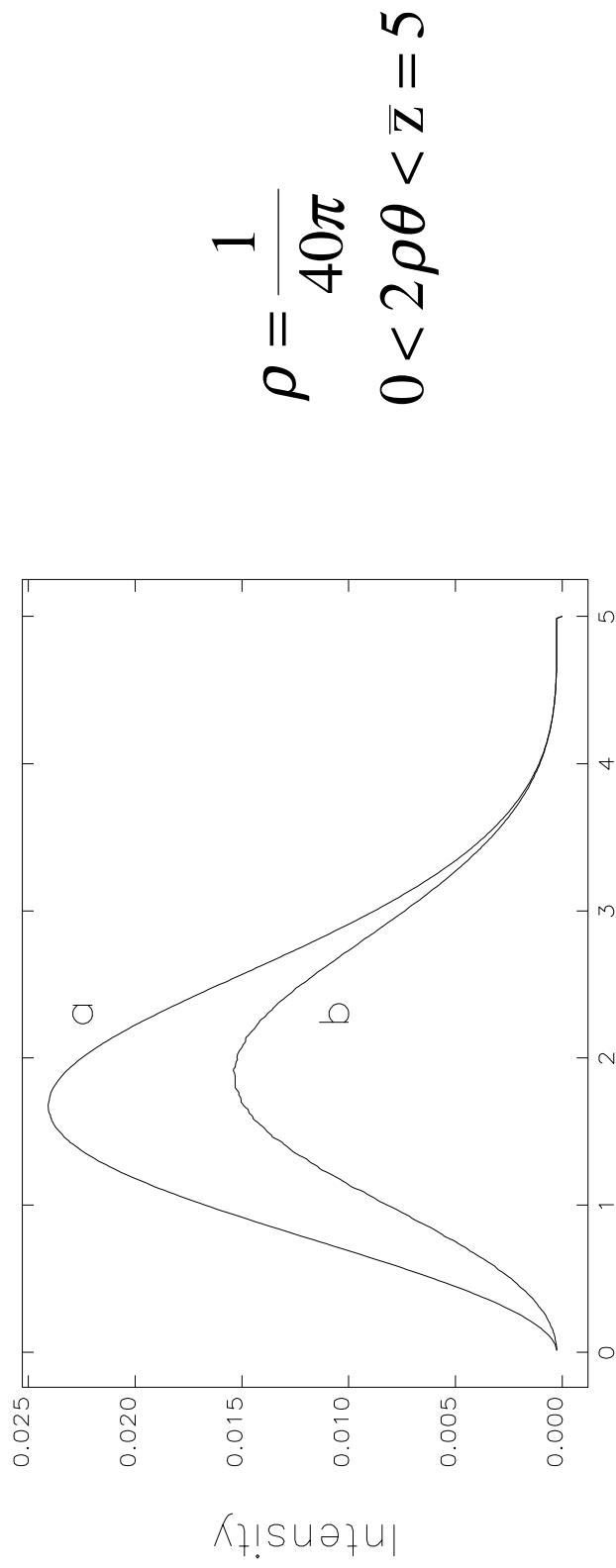
Nonlinear Simulation

- Use the individual particle formulation
- Compute the radiation field by introducing proper particle averaging
 - Follow the time-dependent simulation algorithm (Bonifacio *et al.*)
 - Divide the bunch into buckets (separated by radiation wavelength)
 - Apply steady-state FEL equations in each bucket
 - Slip the field one bucket forward after each wiggler period
 - However, this algorithm cannot “see” the bunch density gradient
 - the Nyquist frequency for the discrete sampling is $c\lambda/2$, below the resonant frequency
- We modify this time-dependent approach
 - by decreasing the sampling interval
 - \Rightarrow cover the resonant frequency
 - \Rightarrow include the CSE effect
- Agree with the multi-frequency CSE simulation by Piovella (1998)
 - Can simulate shot-noise too

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Linear Regime

- Coherent SASE intensity in the slippage region for a flat-top bunch



(a) no initial energy spread, (b) flat-top energy spread of width ρ

- Agrees well with the linear theory

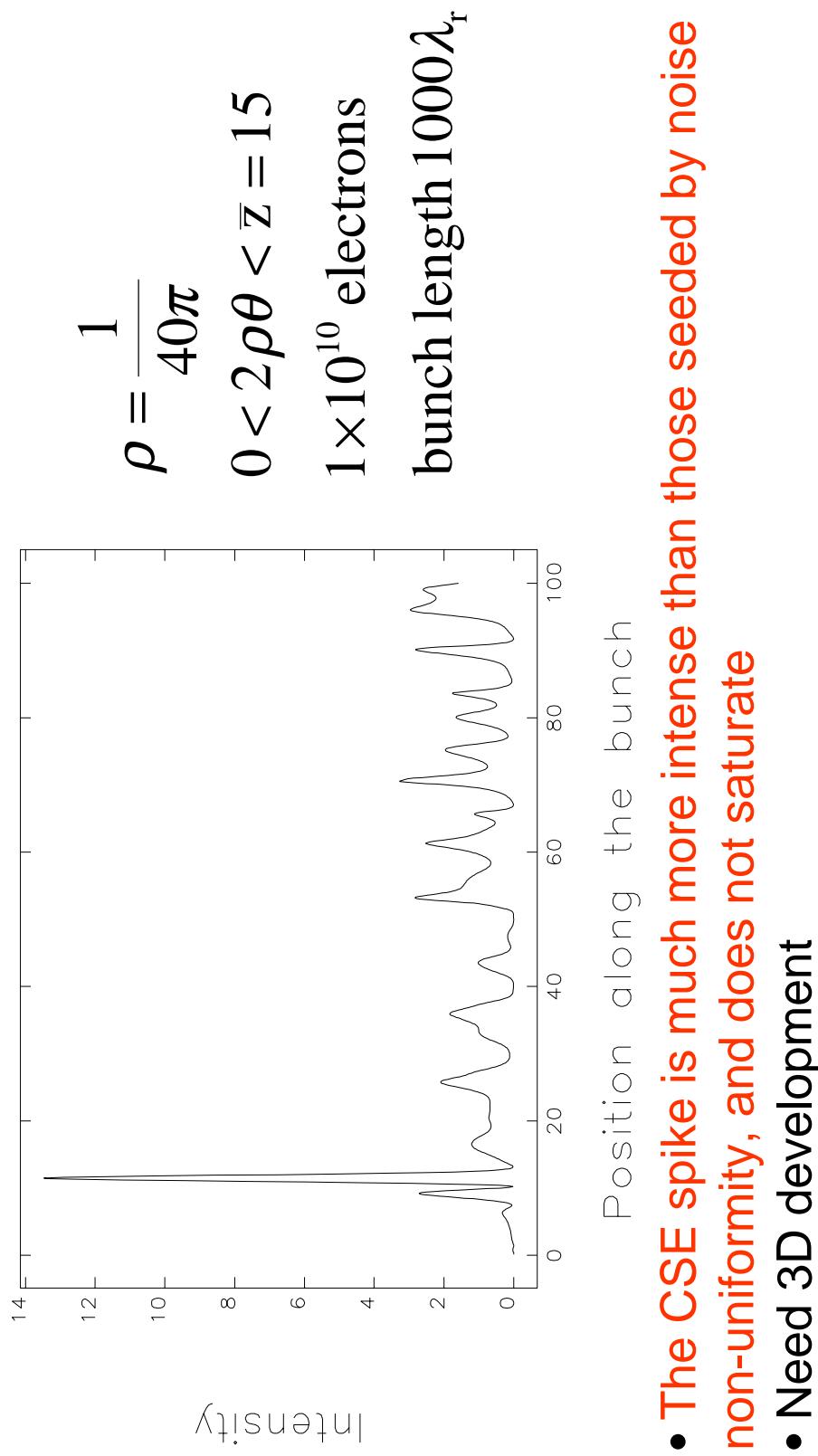
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Zhirong Huang (zrh@aps.anl.gov)

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Nonlinear Regime

- Coherent and incoherent SASE intensity for a flat-top bunch



- The CSE spike is much more intense than those seeded by noise non-uniformity, and does not saturate
- Need 3D development

Transverse Coherence of High-Gain FEL

- In high-gain FEL, it is generally assumed:
a single transv. mode has the largest growth rate ==>
fully transverse coherence can be reached
- Practically important questions:
the transverse profile of the dominant mode and
the (growth rate) separation from its next largest mode
- **Also needed in analysis of the nonlinear harmonic generation
driven by the fundamental radiation**

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Mode Determination

- Paraxial wave equation

$$\left(\frac{\partial}{\partial z} + \frac{\nabla^2}{2ik} \right) E(\mathbf{r}, z) = S(\mathbf{r}, z) \quad (\text{FEL interaction})$$

S is the bunching due to E (electric field)

- TEM_{nm} modes

$$E_{nm}(\mathbf{r}, z) = e^{\lambda_{nm}\bar{z}} R_{nm}(r) e^{im\phi}$$

m is the azimuthal order

n is the radial order

- TEM_{00} mode has the largest growth rate, the dominant mode

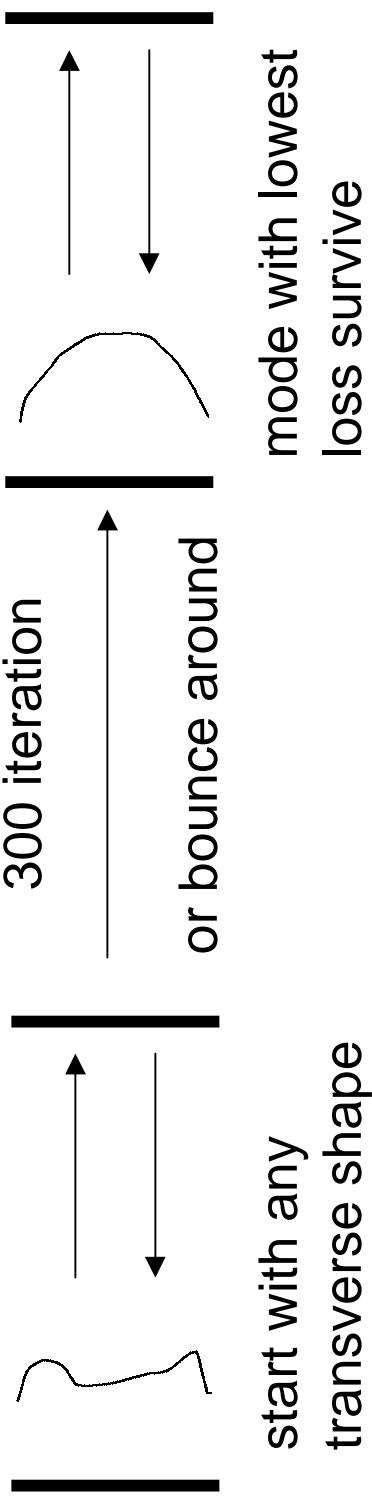
- S term is very complex when emittance and energy spread included

==> Numerical computation using Fox-Li algorithm
(Fox and Li, Bell Sys. Tech. J. 1961)

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Fox-Li Iterative Scheme

- Extensively used in optical cavity design and mode control



start with any
transverse shape

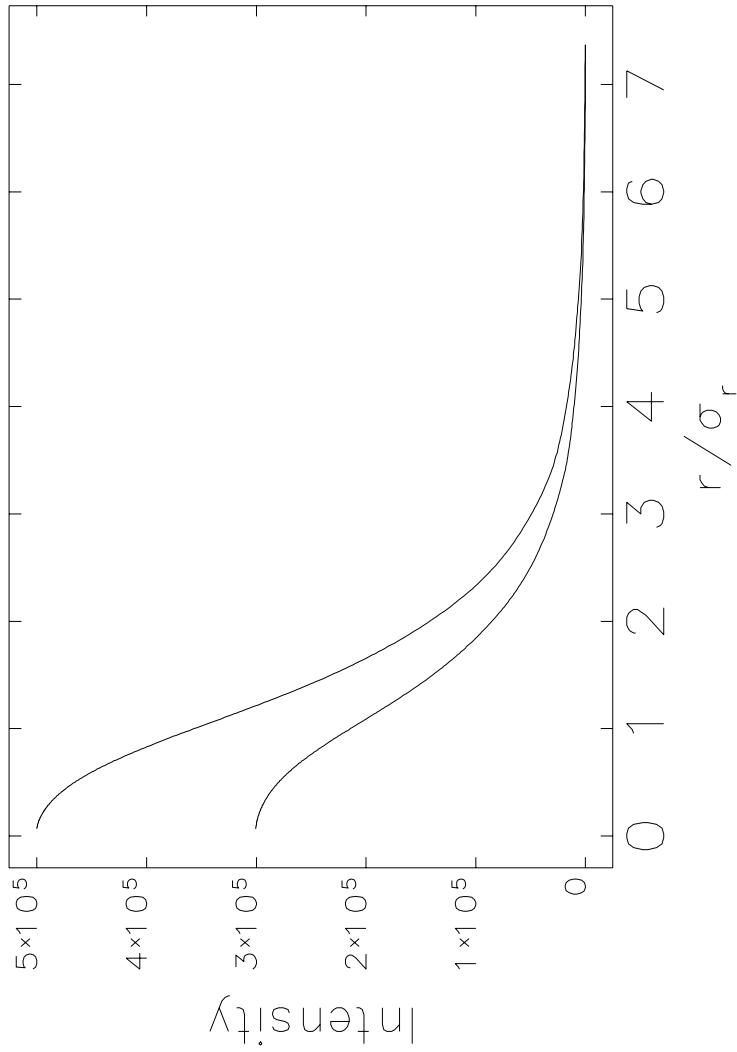
mode with lowest
loss survive

- Analogous to the physical process how a single mode emerges
- In single-pass, high-gain FEL, slice the pass to ~30 steps
- Start with any transverse field profile, apply Fox-Li in successive steps => mode with the highest gain eventually dominates**
- Exact numerical solution**
- “Tricks” can be used to obtain a few higher-order modes

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LEUTL FEL Fundamental Mode

- Using the nominal LEUTL parameter



the self-similar mode at step 29 and 30

- **Mode waist = $2.7 \times \text{e-beam size}$, power gain length = 0.65 m**

Summary and Future Plan

- Obtain the lowest-order mode, the dominant transverse mode
- Reproduce Ming Xie's unpublished work
- Used in the analysis of nonlinear harmonic generation
- Working on the higher-order mode calculation
 $\text{TEM}_{01}, \text{TEM}_{10}$ which has the next shortest gain length?

3D Analysis of Harmonic Generation

- Amplified harmonic emission is possible in a planar wiggler (Colson), but SASE process is heavily in favor of the fundamental frequency
- A 1D model (Bonifacio *et al.*) and a 3D simulation (Freund *et al.*)
=> strong bunching at the fundamental can drive substantial harmonic bunching and sizable harmonic power
- Nonlinear harmonic interaction

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Harmonic Emission and Bunching

- In the electron comoving frame

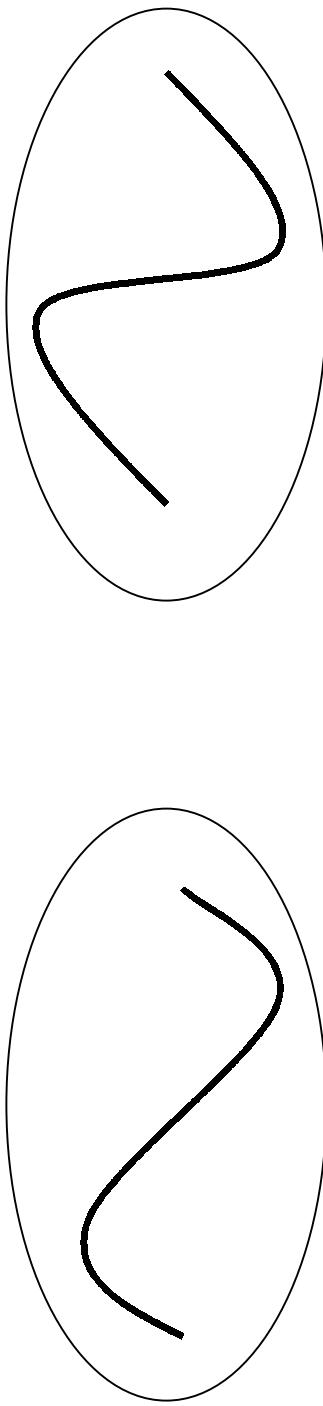
$$8 \longrightarrow E \propto \sum_h a_h(\mathbf{r}, z) e^{ihk_1(z-ct)}$$

$h=1, 3, 5, \dots$ is the (odd) harmonic number
 a_h is the scaled harmonic field

- Before FEL saturation, we assume

$$\dots |a_h| < |a_{h-2}| < \dots < |a_3| < |a_1| < 1$$

- **Harmonics are not independent, in the ponderomotive potential:**



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Harmonic Interaction

- Perturbation to the Vlasov equation to the h^{th} order ==>

$$\left(\frac{\partial}{\partial \bar{z}} + \frac{\nabla^2}{2ih} \right) a_h = \underbrace{a_h}_{\text{linear bunching}} \text{ term} + \underbrace{\sum_{h_1 + \dots + h_m = h} a_{h_1} \times \dots \times a_{h_m}}_{\text{nonlinear harmonic interaction}} \text{ term}$$

- In general

$$a_h = \underbrace{a_h^L}_{\text{SASE}} + \underbrace{a_h^{NL}}_{\text{Nonlinear Harmonic Generation}}$$

- Each a_h^L has a dominant mode with the largest growth rate λ_h
- SASE is predominantly the growth of the fundamental radiation
 $\text{Re}(\lambda_1) > \text{Re}(\lambda_3) > \text{Re}(\lambda_5) > \dots$

- At the fundamental, nonlinear term is much weaker until saturation
- At higher harmonics, nonlinear process may become dominant

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Nonlinear Harmonic Generation

- At the third harmonic, a_3^{NL} is dominated by a_1^3 term

$$a_3^{\text{NL}} \sim a_1^3 \sim \left(\frac{1}{\sqrt{N_c}} e^{\lambda_1 \bar{z}} \right)^3 \quad \text{and} \quad a_3^L \sim \frac{1}{\sqrt{N_c}} e^{\lambda_3 \bar{z}} \quad (N_c \text{ noise})$$

$\Rightarrow a_3^{\text{NL}} \gg a_3^L$ at some stage of the exponential gain regime

- At the fifth, a_5^{NL} is dominated by $a_1^2 a_3$ and a_1^5
- $\Rightarrow a_5^{\text{NL}} \sim a_1^5$ can be larger than a_5^L before saturation
- In general, $a_h^{\text{NL}} \sim a_1^h$, with a growth rate $h\lambda_1$ (Bonifacio..., Freund...)

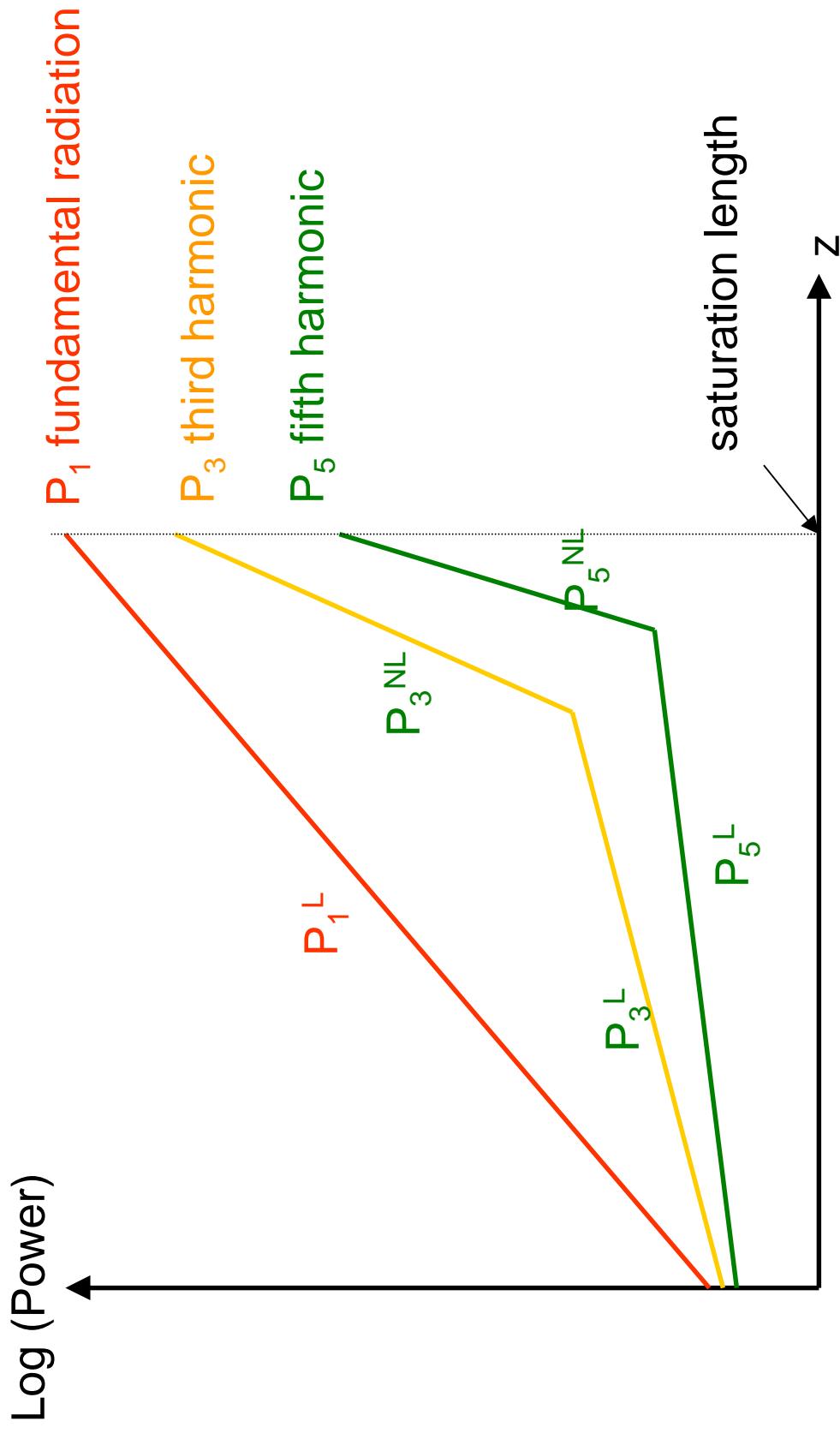
- The transverse profiles of the nonlinear harmonics are completely determined by the fundamental radiation profile (mode)

$$\boxed{\text{Bandwidth of } a_h^{\text{NL}} = \frac{1}{h} \times \text{Bandwidth of } a_1}$$

- Coherence length of a_h^{NL} is the same as the fundamental

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Evolution of Harmonic Radiation



Third Harmonic Radiation

- Assume the initial coasting beam is Gaussian in transverse coordinates and in energy variables
- Assume a Gaussian mode for the transverse profile of the fundamental radiation (with the predetermined waist)
- **Calculation takes into account all 3D effects such as emittance, energy spread, diffraction, and guiding**
- **Arrive at a set of equations easy for numerical computation**
- For example
LEUTL FEL: power at the third harmonic (at 180 nm)

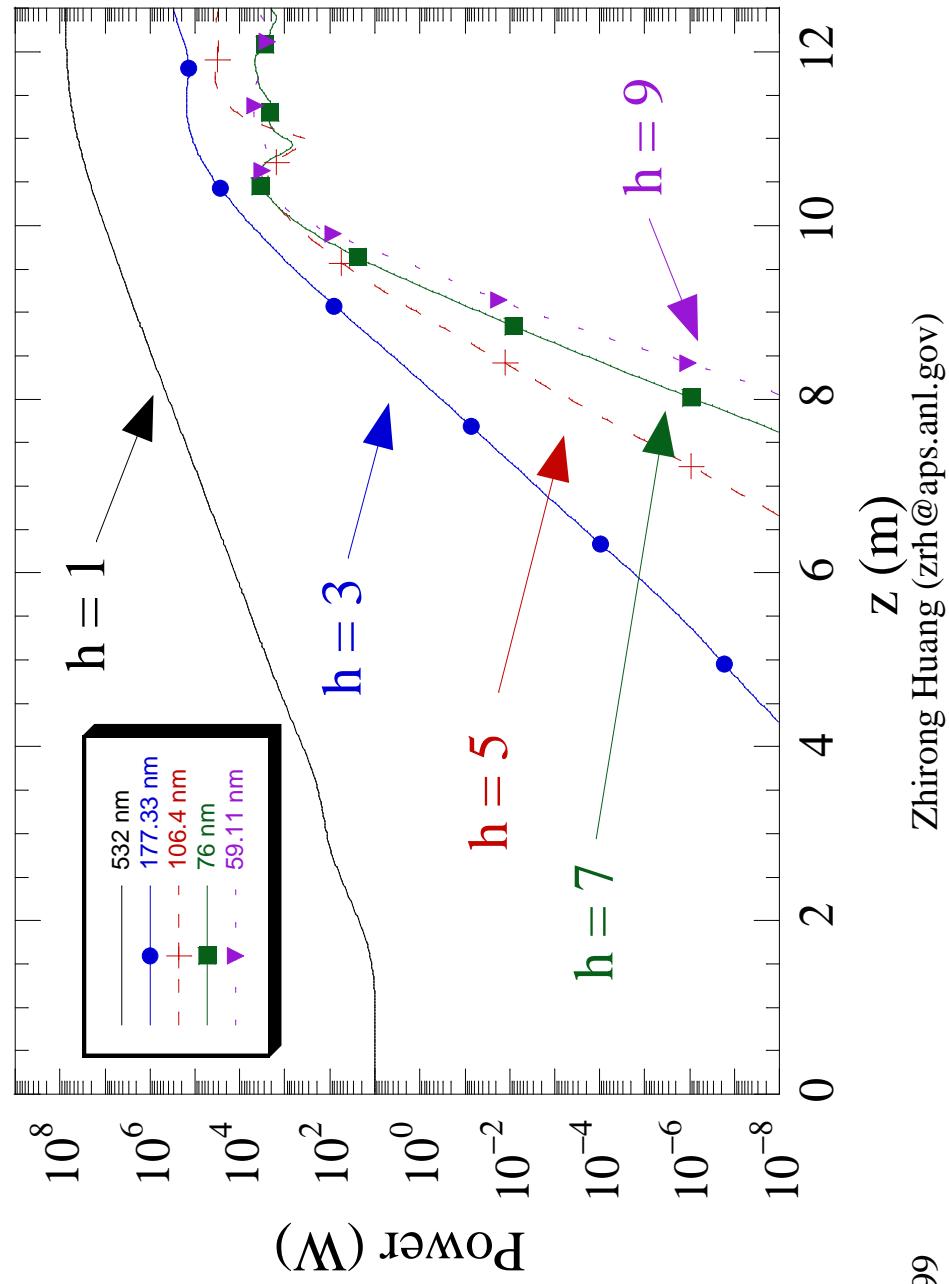
$$\frac{P_3^{NL}}{\rho P_{\text{beam}}} = 0.017 \left(\frac{P_1^{NL}}{\rho P_{\text{beam}}} \right)^3 \quad (P_{\text{beam}} \text{ is the beam power})$$

- => **right before saturation, $P_3 \sim 1\% P_1$**
For LCLS, similar power level exists (third harmonic at 0.5 Å)
- **Better brightness**

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MEDUSA simulation using a single-segmented wiggler
for the nominal LEUTL parameter (S. Biedron at FEL99)

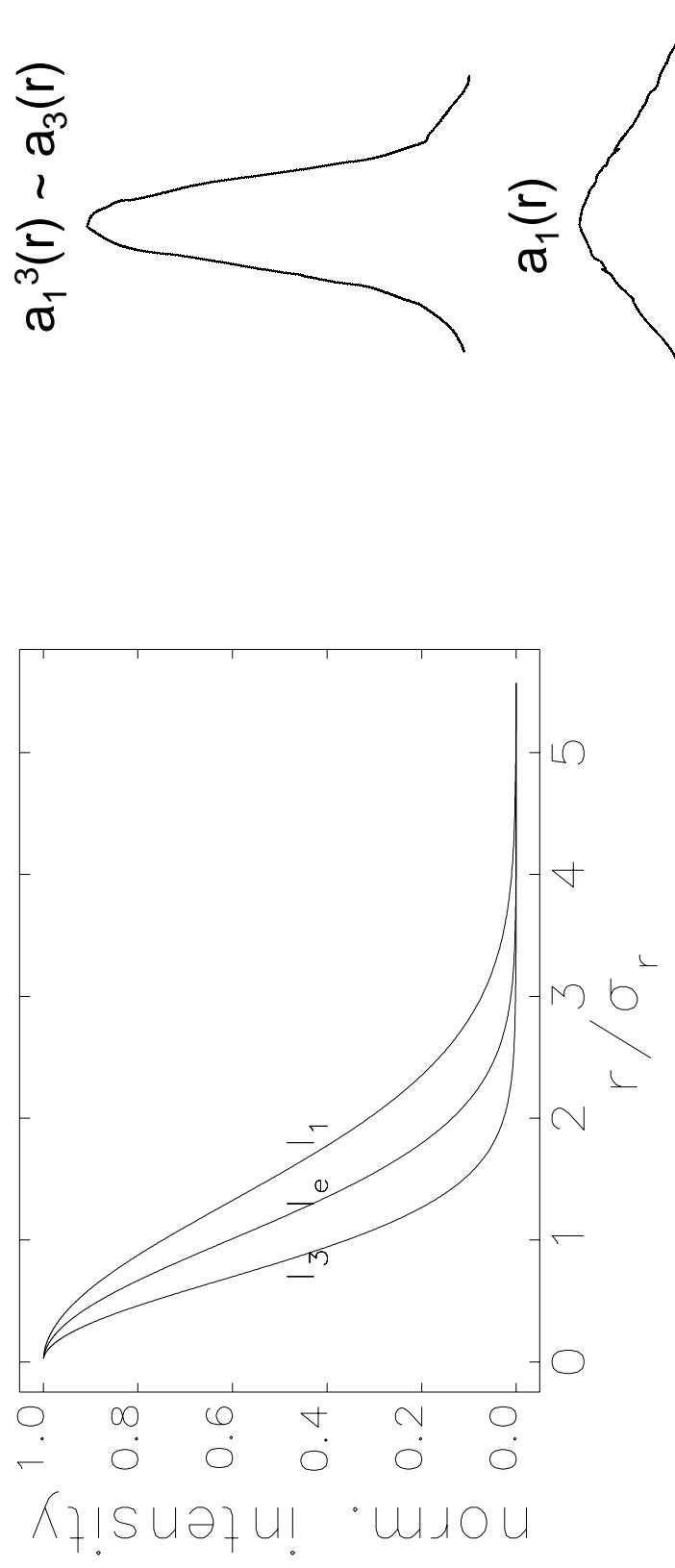
ODD HARMONICS OF 532nm



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Transverse Profile

- Numerical calculation of the third harmonic intensity for LEUTL FEL



- The optical mode of the third harmonic is also guided, with a narrower waist than the fundamental due to the nonlinear mechanism

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Zhirong Huang (zrh@aps.anl.gov)

Conclusions

- Nonlinear harmonic interaction exists in any high-gain FEL, not only in SASE FELs
- Plan to measure the third harmonic (at $1.7 \mu\text{m}$) of the HGHG experiment (S. Biedron *et al.*) to verify the theoretical prediction
- In addition to other harmonic generation schemes (Bonifacio, Yu), the nonlinear harmonic interaction could be a promising mechanism to generate coherent radiation at short wavelengths